Dynamic Loading

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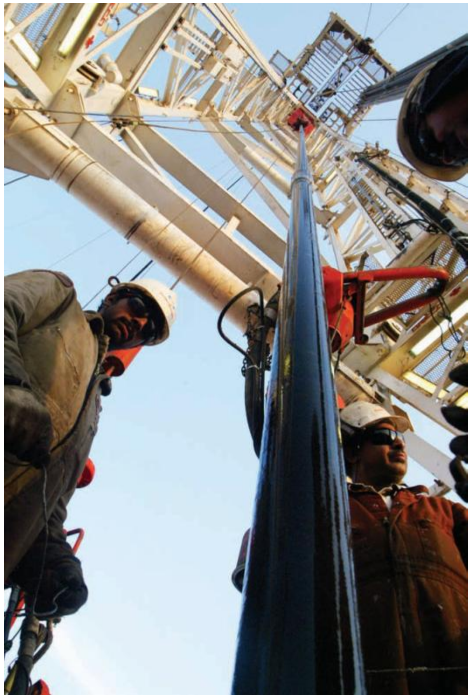
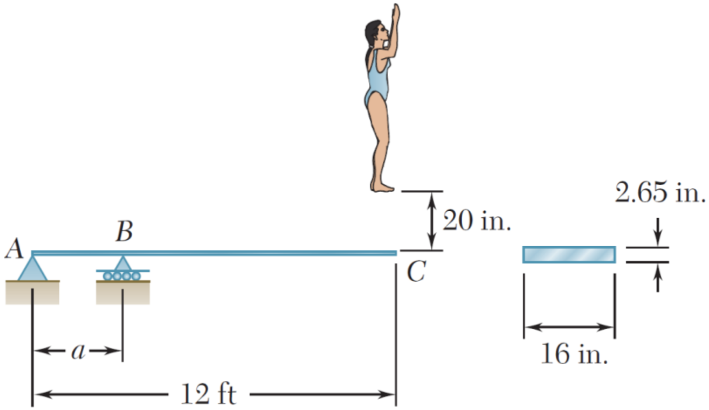
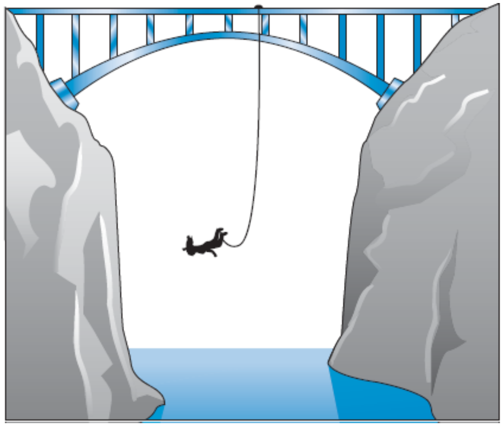
Static vs. Dynamic Loading

- **Static Loading:** external loads are gradually increased to a definite value and subsequently are held in constant.



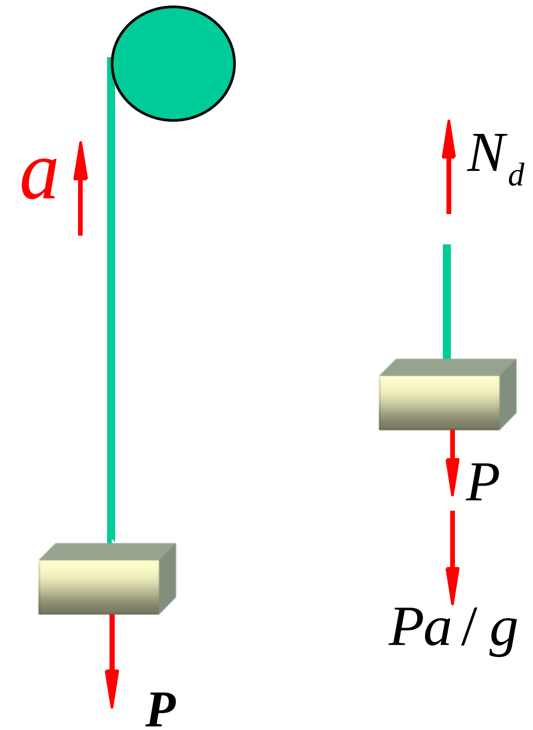
- **Dynamic Loading:** the magnitude and/or direction of external loads vary as a function of time. This type of loads generate significant acceleration within structural members that cannot be neglected.





Structural Members under Constant Acceleration

- Given: P, A and a.
- Find: the stress developed in the steel rope.
- Solution by the **method of statnamic**:



1. Inertia force: $F = \dfrac{P}{g}a$

2. Axial force in rope: $N_d = P + \dfrac{P}{g}a = P\left(1 + \dfrac{a}{g}\right)$

3. Normal stress developed in rope:

$$\sigma_d = \frac{N_d}{A} = \frac{P}{A}(1 + \frac{a}{g}) = \sigma_{st}\left(1 + \frac{a}{g}\right)$$

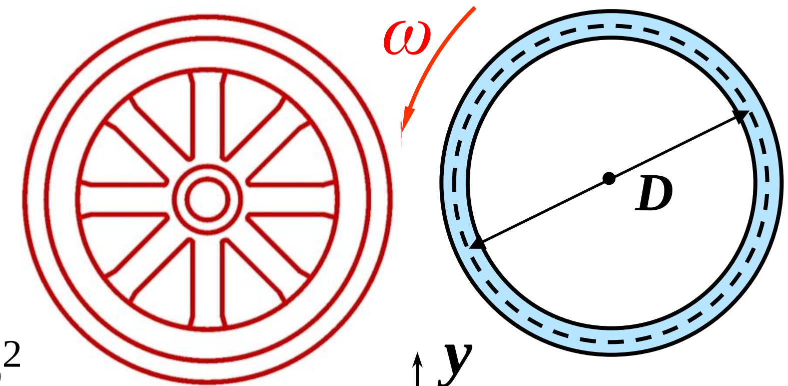
4. Dynamic load factor: $k_d = \dfrac{N_d}{N_{st}} = \dfrac{\sigma_d}{\sigma_{st}} = \dfrac{\Delta_d}{\Delta_{st}} = \left(1 + \dfrac{a}{g}\right)$

Structural Members under Constant Rotation

- Given: ω, A, ρ and D.
- Find: the stress developed in the flange of the flying wheel.
- Solution by the **method of statnamic**:

1. Inertia force acting on unit length:

$$q_d = \rho \cdot 1 \cdot A \cdot \omega^2 \frac{D}{2}$$

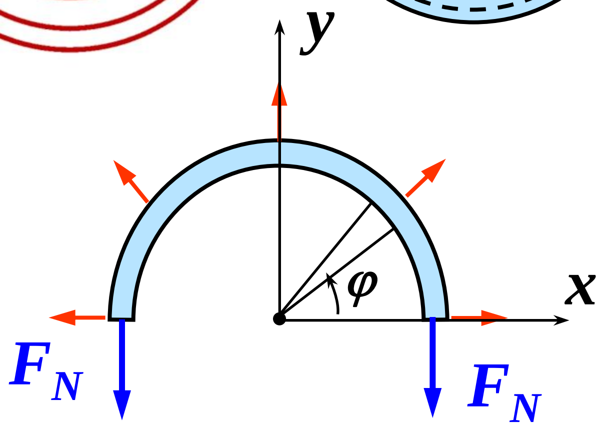


2. Circumferential force:

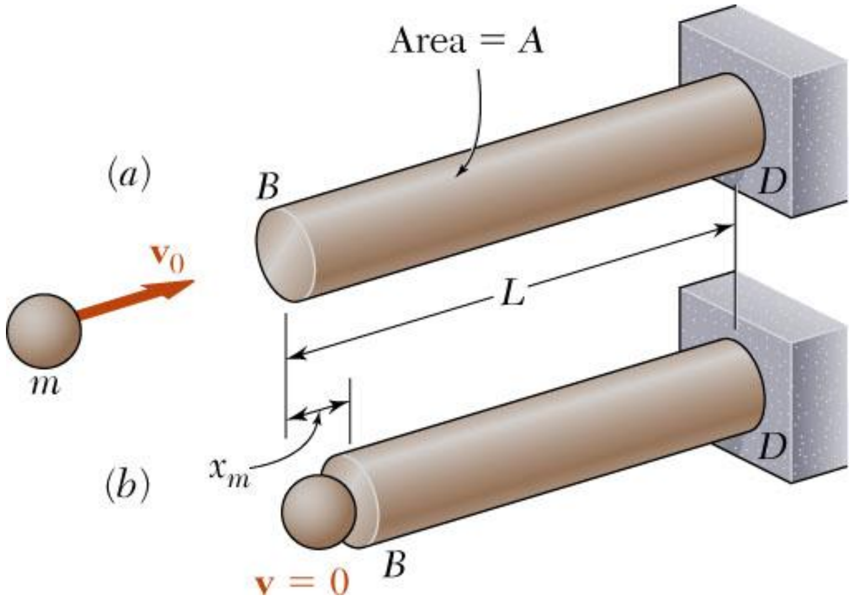
$$F_N = \frac{1}{2}\int_0^{\pi} q_d \cdot \frac{D}{2} d\varphi \sin\varphi = \frac{\rho A D^2 \omega^2}{4}$$

3. Circumferential stress:

$$\sigma_d = \frac{F_{Nd}}{A} = \frac{\rho \omega^2 D^2}{4}$$



Horizontal Impact on Axial Members



- **Horizontal Impact**: Consider a rod which is hit at its end with a body of mass *m* moving with a velocity $v_{0.}$
- Rod deforms under impact. Stresses reach a maximum value σ_{max} and then disappear.

- To determine the maximum stress σ_{max}
  - Assume that the kinetic energy is transferred entirely to the structure,

$$U = \tfrac{1}{2} m v_0^2$$

  - Assume that the stress-strain diagram obtained from a static test is also valid under impact loading.

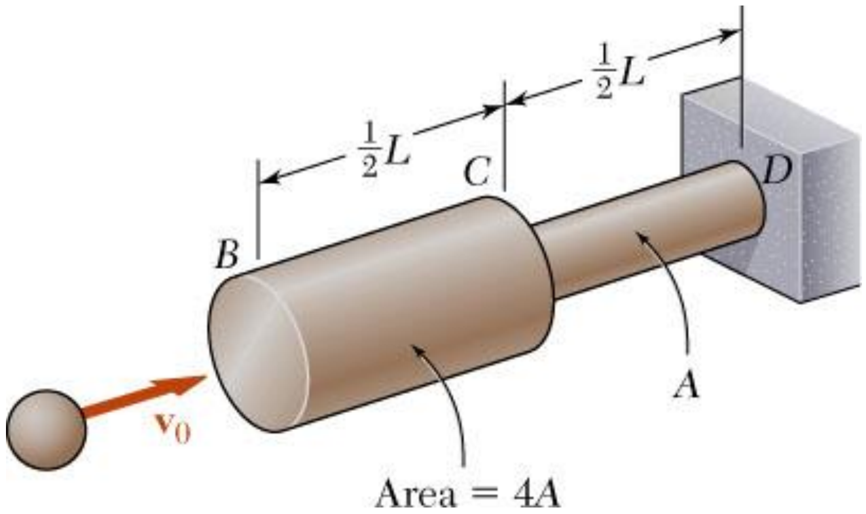
- Energy conservation requires,

$$U = \int \frac{\sigma_{max}^2}{2E} dV = \int \frac{P^2}{2EA} dx$$

- For the case of a uniform rod,

$$U = \frac{P^2 L}{2EA} \Rightarrow P = \sqrt{\frac{2UAE}{L}} = \sqrt{\frac{m v_0^2 AE}{L}}$$

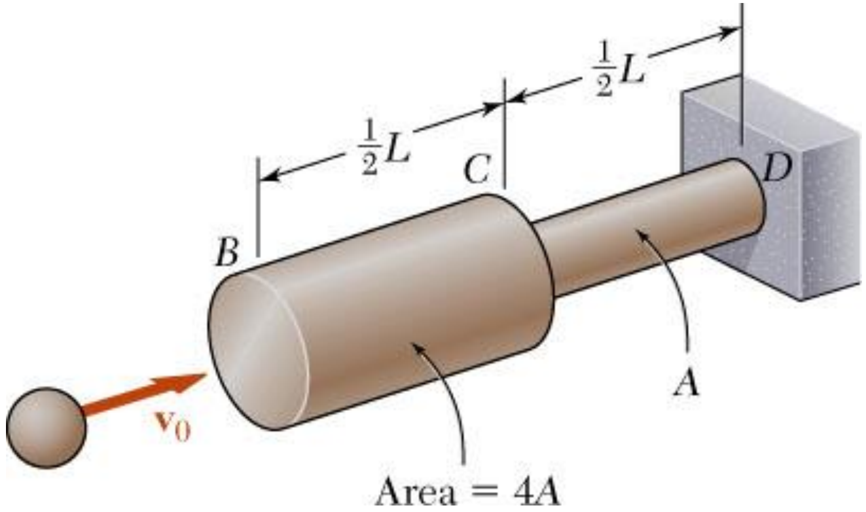
Sample Problem



Horizontal Impact: Body of mass *m* with velocity v_0 hits the end of the nonuniform rod *BCD*. Knowing that the diameter of the portion *BC* is twice the diameter of portion *CD*, determine the maximum value of the normal stress in the rod.

Solution:

- Due to the change in diameter, the normal stress distribution is nonuniform.
- Find the static load *P* which produces the same strain energy as the impact.
- Evaluate the maximum stress resulting from the static load *P*



Solution:

- Due to the change in diameter, the normal stress distribution is nonuniform.

$$U = \tfrac{1}{2} m v_0^2$$

$$= \int \frac{\sigma^2}{2E} dV = \frac{P^2 L}{2EA}$$

- Find the static load P which produces the same strain energy as the impact.

$$U = \frac{P^2 (L/2)}{2EA} + \frac{P^2 (L/2)}{2E(4A)} = \frac{5}{16} \frac{P^2 L}{EA}$$

$$P = \sqrt{\frac{16}{5} \frac{UEA}{L}} = \sqrt{\frac{8}{5} \frac{m v_0^2 EA}{L}}$$

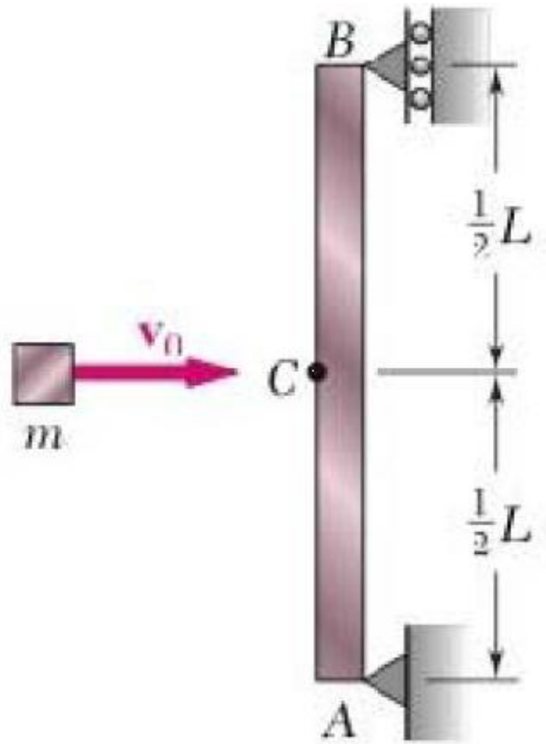
- Evaluate the maximum stress resulting from the static load P

$$\sigma_{\max} = \frac{P}{A}$$

$$= \sqrt{\frac{16}{5} \frac{UE}{AL}}$$

$$= \sqrt{\frac{8}{5} \frac{m v_0^2 E}{AL}}$$

Horizontal Impact on Beams



Horizontal Impact: A block of mass *m* moving with a velocity v_0 hits squarely the prismatic member *AB* at its midpoint *C*. Determine (*a*) the equivalent static load *P*, (*b*) the maximum stress σ_{max} in the member, and (c) the maximum deflection Δ_{max} at *C*.

Solution:

- Kinetic energy of the block.

$$U_m = mv_0^2/2$$

- Find the static load *P* which produces the same strain energy as the impact. For loaded beam as shown,

$$\begin{cases} U = \dfrac{1}{2}P\Delta = \dfrac{1}{2}\dfrac{P^2L^3}{48EI} \\ P = \sqrt{\dfrac{96EIU}{L^3}} = \sqrt{\dfrac{48mv_0^2EI}{L^3}} \end{cases}$$

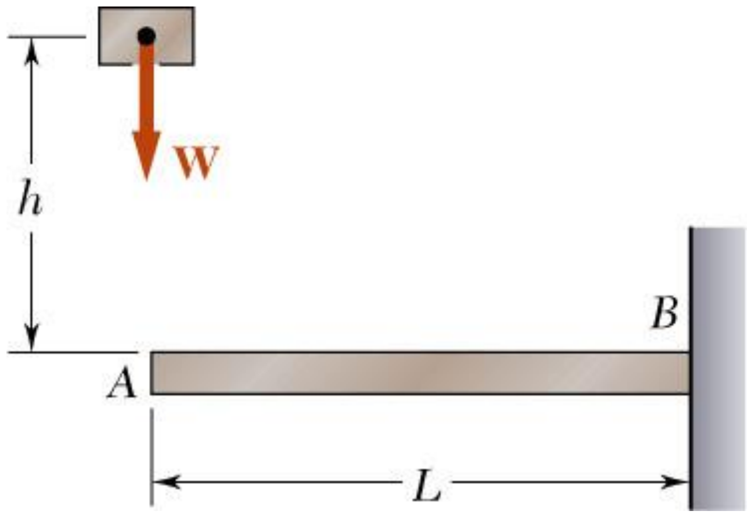
- Maximum stress

$$\sigma_{\max} = \frac{M_{\max}c}{I} = \frac{PLc}{4I} = \sqrt{\frac{3mv_0^2EI}{(I/c)^2L}}$$

- Maximum deflection

$$\Delta_{\max} = \frac{PL^3}{48EI} = \sqrt{\frac{mv_0^2L^3}{48EI}}$$

Vertical Impact on Beams



Vertical Impact: A block of weight *W* is dropped from a height *h* onto the free end of the cantilever beam. Determine the maximum value of the stresses in the beam.

Solution:

- Work done by the impact load.

$$U = W\left(h + \Delta_d\right)$$

- Find the static load *P* which produces the same strain energy as the impact.
- Treat the beam as a spring:

$$U = \frac{1}{2}P\Delta_d = \frac{P^2}{2k} = \frac{1}{2}k\Delta_d^2$$

- Energy conservation requires,

$$\frac{1}{2}k\Delta_d^2 = W\left(h + \Delta_d\right)$$

$$\Rightarrow \frac{1}{2}\Delta_d^2 = \frac{W}{k}\left(h + \Delta_d\right) = \Delta_{st}\left(h + \Delta_d\right)$$

$$\Rightarrow \Delta_d^2 - 2\Delta_{st}\Delta_d - 2\Delta_{st}h = 0$$

Vertical Impact on Beams

$$\Rightarrow \Delta_d = \frac{2\Delta_{st} \pm \sqrt{(2\Delta_{st})^2 + 8\Delta_{st}h}}{2} = \Delta_{st} \pm \sqrt{\Delta_{st}^2 + 2\Delta_{st}h} = \left(1 \pm \sqrt{1 + 2h/\Delta_{st}}\right)\Delta_{st}$$

$$\Rightarrow \Delta_d = \left(1 + \sqrt{1 + 2h/\Delta_{st}}\right)\Delta_{st} = K_d\Delta_{st}$$

Dynamic load factor: $K_d = 1 + \sqrt{1 + 2h/\Delta_{st}}$

For the present problem:

$$\Delta_{st} = \frac{WL^3}{3EI}, \quad \sigma_{st\cdot\max} = \frac{M_{\max}y_{\max}}{I} = \frac{WLc}{I}$$

$$\Rightarrow \Delta_d = K_d\Delta_{st} = \left(1 + \sqrt{1 + \frac{6EIh}{WL^3}}\right)\Delta_{st} = \frac{PL^3}{3EI}$$

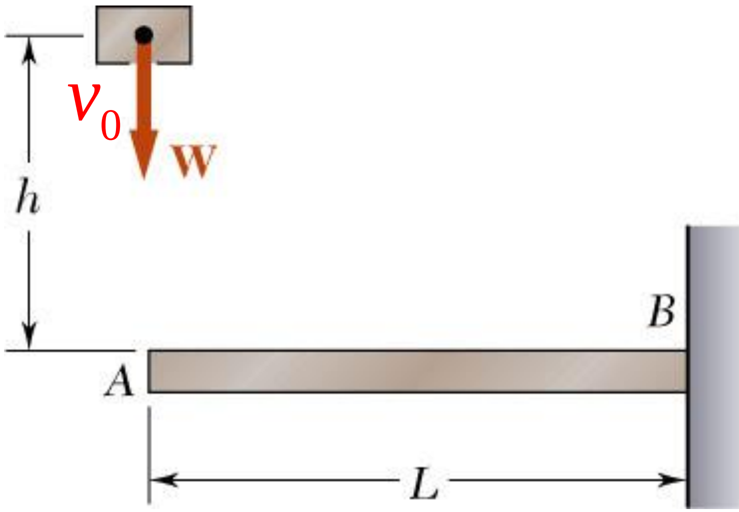
$$\Rightarrow P = K_dW = \left(1 + \sqrt{1 + \frac{6EIh}{WL^3}}\right)W$$

$$\Rightarrow \sigma_{d\cdot\max} = K_{st\cdot\max}\sigma_{st} = \left(1 + \sqrt{1 + \frac{6EIh}{WL^3}}\right)\frac{WLc}{I}$$

For sudden loading $(h = 0) \Rightarrow K_d = 2$

For $h >> \Delta_{st} \Rightarrow \begin{cases} K_d = \sqrt{2h/\Delta_{st}} \approx \sqrt{\dfrac{6EIh}{WL^3}} \\ \Delta_d = K_d\Delta_{st} \approx \sqrt{\dfrac{2hWL^3}{3EI}}, \\ P = K_dW \approx \sqrt{\dfrac{6EIhW}{L^3}} \\ \sigma_{d\cdot\max} = K_d\sigma_{st\cdot\max} \approx \sqrt{\dfrac{6EhWc^2}{IL}} \end{cases}$

Vertical Impact with an Initial Velocity



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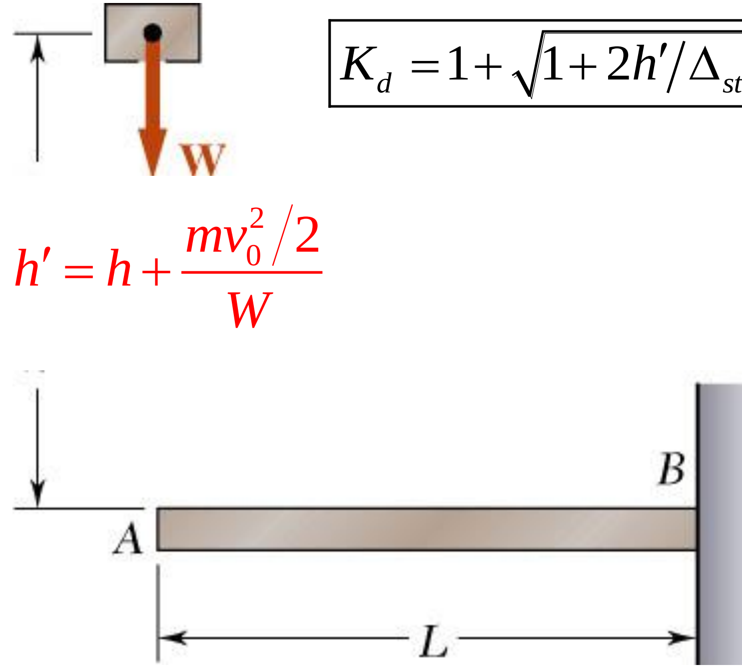
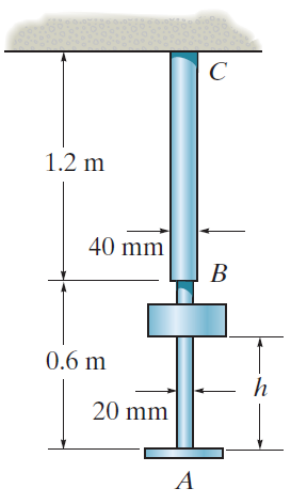
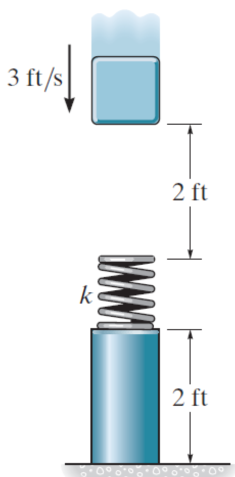


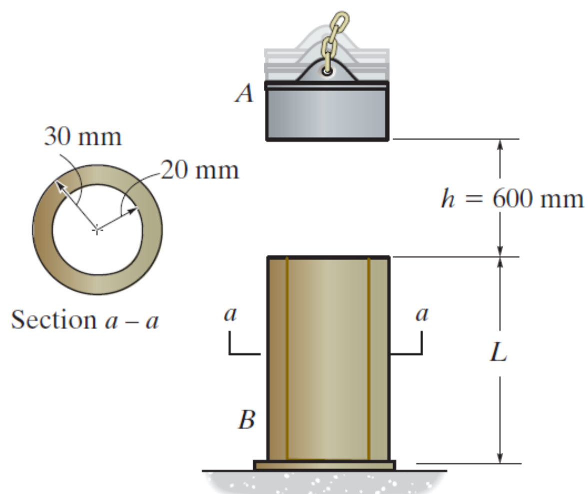
$$K_d = 1 + \sqrt{1 + 2h'/\Delta_{st}}$$

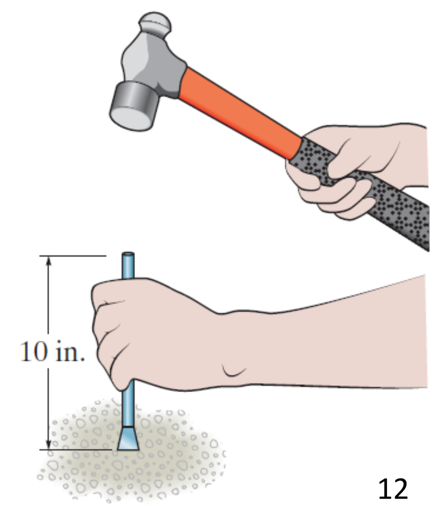
$$h' = h + \frac{m v_0^2 / 2}{W}$$

- **Further Examples:**









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