1. Using index notation, explicitly verify the following identities:

   a. $\nabla^2(\phi\psi) = (\nabla^2\phi)\psi + \phi(\nabla^2\psi) + 2\nabla\phi\cdot\nabla\psi$

   b. $\nabla\times(\phi\mathbf{u}) = \nabla\phi\times\mathbf{u} + \phi(\nabla\times\mathbf{u})$

   c. $\nabla\cdot(\mathbf{u}\times\mathbf{v}) = \mathbf{v}\cdot(\nabla\times\mathbf{u}) - \mathbf{u}\cdot(\nabla\times\mathbf{v})$

   d. $\nabla\times(\nabla\times\mathbf{u}) = \nabla(\nabla\cdot\mathbf{u}) - \nabla^2\mathbf{u}$

   e. $\mathbf{u}\times(\nabla\times\mathbf{u}) = \frac{1}{2}\nabla(\mathbf{u}\cdot\mathbf{u}) - \mathbf{u}\cdot\nabla\mathbf{u}$

2. a. Show that the volume V of the region bounded by a closed surface S is given by

$$V = \frac{1}{3}\int_S (\mathbf{x}\cdot\mathbf{n})\, dS\,,$$

where $\mathbf{n}$ is the outward unit normal to S and $\mathbf{x}$ is the position vector in V.

b. For a constant vector $\mathbf{a}$ (independent of position $\mathbf{x}$), prove that

$$2\mathbf{a}V = \int_S (\mathbf{n}\times\mathbf{a}\times\mathbf{x})\, dS\,.$$

3. Verify Green's theorem for the line integral

$$\oint_C \left[\left(xy + y^2 \right) dx + x^2 dy \right],$$

where C is the closed curve formed by $y = x^2$ and $y = x$.

4. Determine the forms of $\nabla\phi$, $\nabla^2\phi$, $\nabla\cdot\mathbf{u}$, $\nabla\times\mathbf{u}$ for three-dimensional cylindrical and spherical coordinates.