

1. In terms of principal stress state $(\sigma_a, \sigma_b, \sigma_c)$, derive the normal and shear stress on the octahedral plane in the first quadrant (whose surface normal forms equal angles with each of the three principal axes).

2. For a continuous medium, derive the body forces needed for equilibrium due to the following stress state:

$$\sigma_x = x^2 + y^2, \quad \sigma_z = x^2 + z^2, \quad \tau_{xy} = xy, \quad \sigma_y = \tau_{xz} = \tau_{yz} = 0.$$

3. Show that the following stress components satisfy the equations of equilibrium with zero body forces, but are not the solution to a problem in elasticity, i.e. not satisfying the compatibility equations:

$$\begin{aligned}\sigma_x &= y^2 + \nu(x^2 - y^2), & \sigma_y &= x^2 + \nu(y^2 - x^2), \\ \sigma_z &= \nu(x^2 + y^2), & \tau_{xy} &= -2\nu xy, & \tau_{xz} &= \tau_{yz} = 0.\end{aligned}$$

4. Assuming zero body forces, derive the most general expressions for σ_y and τ_{xy} from the equilibrium and compatibility conditions, provided that

$$\sigma_x = \sigma_z = \tau_{xz} = \tau_{yz} = 0.$$